Gems of TCS

Data Structures

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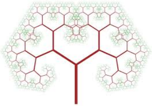
September 5, 2023

Data Structures

Stack, Queue, List, Heap



Search Trees

```
hash(unsigned x) {
   x ^= x >> (w-m);
   return (a*x) >> (w-m);
}
```

Hash Tables

Coping with Hard Problems

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- Today: Preprocessing

EXAMPLES

- Graph Distances: Preprocess a road network in order to efficiently compute distance queries between cities
  (Google Maps)

Examples

- Graph Distances: Preprocess a road network in order to efficiently compute distance queries between cities
  (Google Maps)
- Clustering: Preprocess a set of movies in order to efficiently find closest movie to a query movie
  (Netflix recommendations)

Data Structures

Preprocessing

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Queries

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New York — Washington

Preprocessing

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Queries

New York — Washington

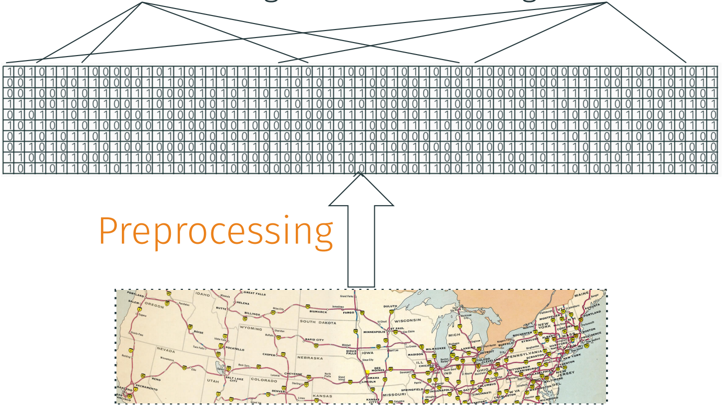
Preprocessing

Data Structures

Queries

New York — Washington

Washington — Boston



Stealing Passwords

Password Hashing

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haveibeenpwned.com:
Your account has
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hash(qwerty)=1xe4ht

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- Ideally, to find a password that leads to a fixed hash value, one needs to brute force all possible passwords
- Hash functions are publicly known (SHA-3)
- For now, consider hash functions $f\colon \{1, \ldots, N\} \to \{1, \ldots, N\}$ that are bijections

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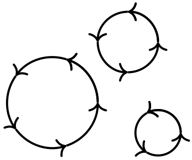
- Thus, this graph is a union of cycles



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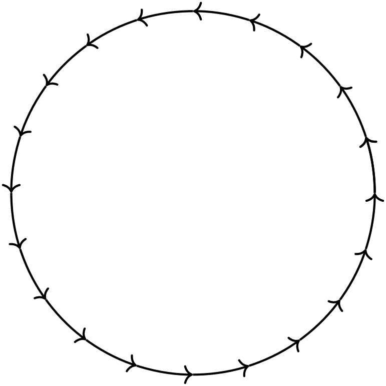

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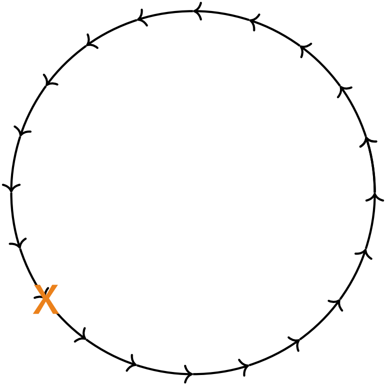

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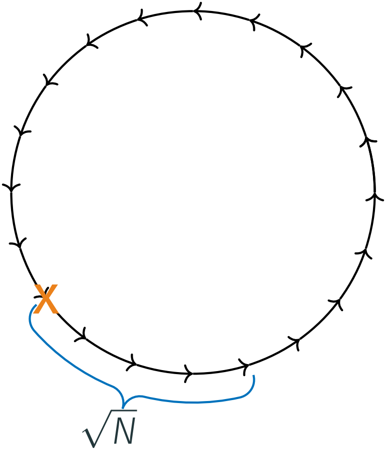

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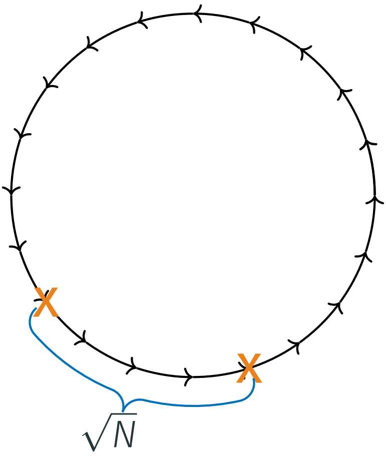

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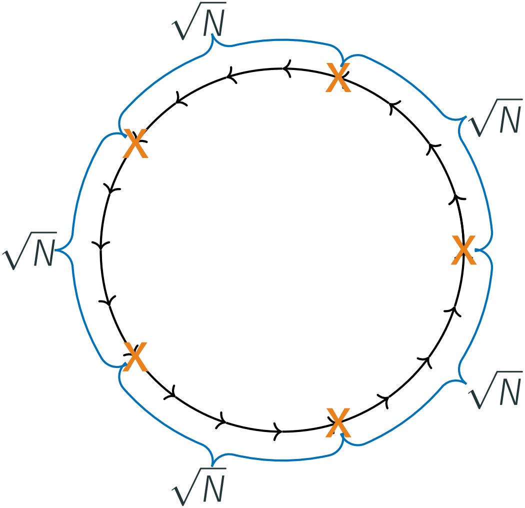

Inverting a Bijection

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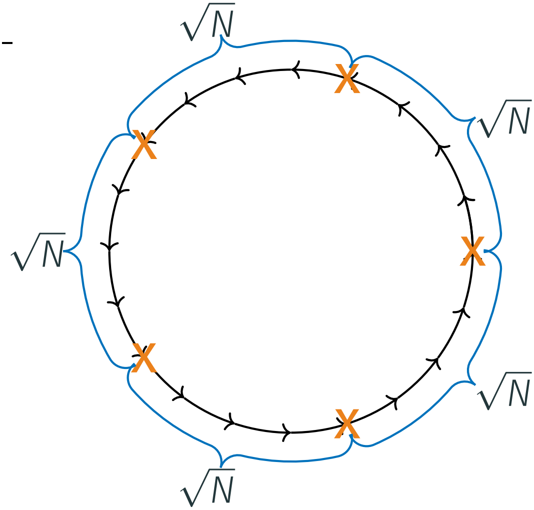

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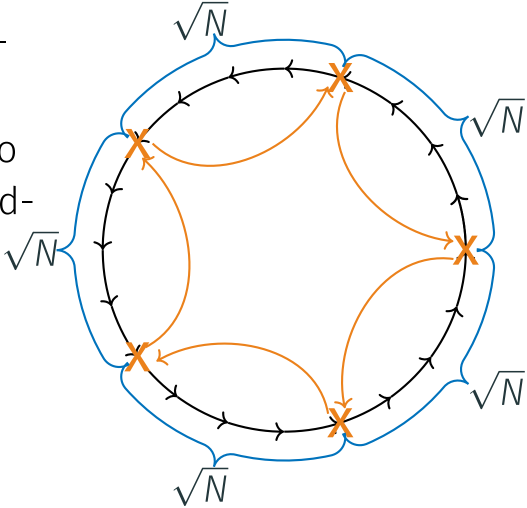

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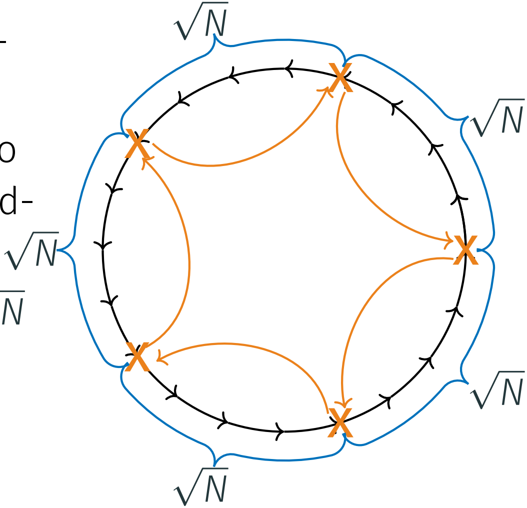

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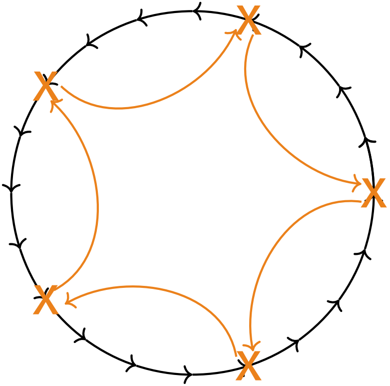

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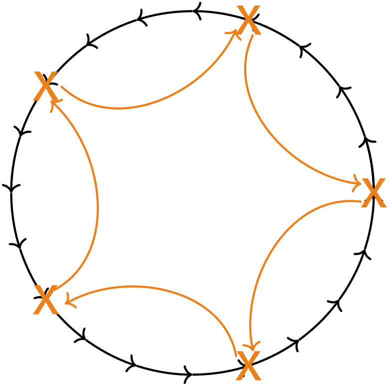

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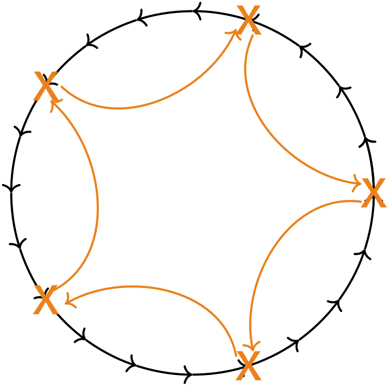

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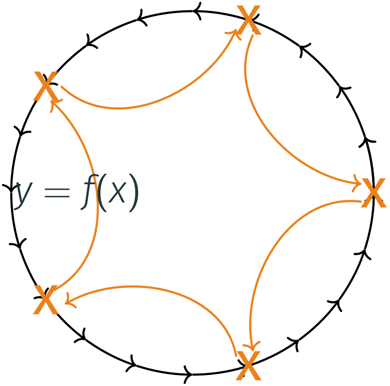

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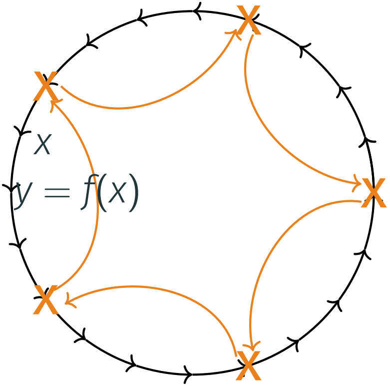

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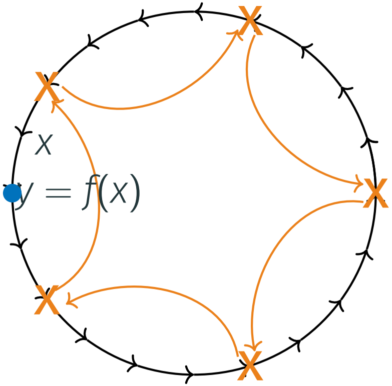

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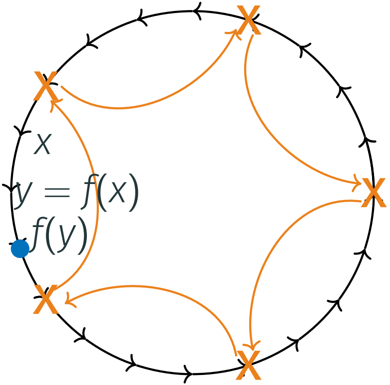

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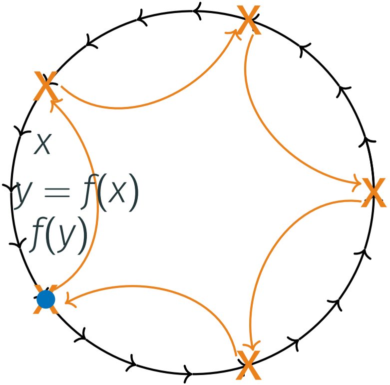

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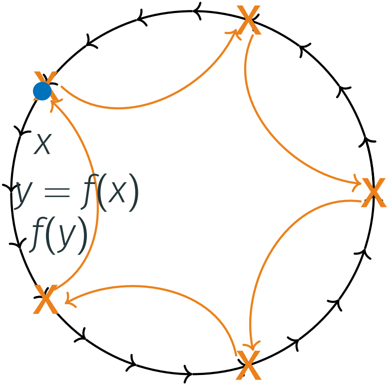

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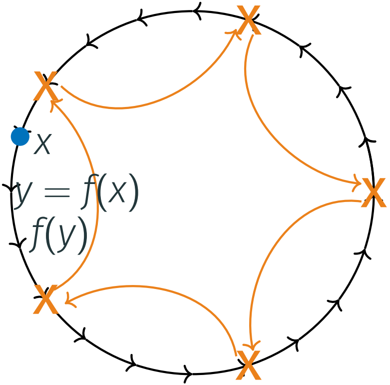

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- Space: S, query time: T

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- We'll be wrong with small probability

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- We'll use $k = O(1)$ hash functions

Hash Functions

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- Assume that functions are independent and uniform random

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  - for $i = 1, \ldots, k$,
    - $A[f_i(x)] \leftarrow 1$
- Lookup(x):
  - return 1 iff for every $i = 1, \ldots, k$, $A[f_i(x)] = 1$

Analysis