EVERYTHING IS A BIT STRING

• Input to an algorithm is a string
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• Algorithm itself is a string
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• Given input, algorithm
Everything is a Bit String

- Input to an algorithm is a string
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- Every string is an algorithm
- Given input, algorithm
  - either eventually outputs some value
EVERYTHING IS A BIT STRING

- Input to an algorithm is a string
- Algorithm itself is a string
- Every string is an algorithm

- Given input, algorithm
  - either eventually outputs some value
  - or never halts
Halting Problem
INFINITE LOOPS

```
    i = 0
    while i <= 5:
        print('Infinite loop')
```
INFINITE LOOPS

```python
i = 0
while i <= 5:
    print('Infinite loop')

x = True
while x:
    print('Infinite loop')
```
HALTING PROBLEM

- Function HALT is defined as follows.
Halting Problem

• Function HALT is defined as follows.
  • The first input is algorithm $A$
  • HALT($A$, $x$) = 1 if $A$ halts on input $x$
  • HALT($A$, $x$) = 0 if $A$ enters infinite loop on input $x$
Halting Problem

• Function HALT is defined as follows.
  • The first input is algorithm A
  • The second input is string x
Halting Problem

- Function HALT is defined as follows.
  - The first input is algorithm $A$
  - The second input is string $x$
  - $\text{HALT}(A, x) = 1$ if $A$ halts on input $x$
Function HALT is defined as follows.

- The first input is algorithm \( A \)
- The second input is string \( x \)
- \( \text{HALT}(A, x) = 1 \) if \( A \) halts on input \( x \)
- \( \text{HALT}(A, x) = 0 \) if \( A \) enters infinite loop on input \( x \)
Applications of Halting Problem

- Algorithm for HALT will help to design bug-free soft (and hardware)

- Goldbach's conjecture
- Collatz conjecture
- Twin (cousin/sexy) prime conjecture
- Odd perfect number
APPLICATIONS OF HALTING PROBLEM

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• Algorithm for HALT will (eventually) solve many mathematical problems
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  • Goldbach’s conjecture
  • Collatz conjecture
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  • Odd perfect number
  • ...
Clearly, every function can be computed given sufficient time.
Except this is not true
HALTING IS UNDECIDABLE
• Easy to solve for one input and one algorithm
Remarks

• Easy to solve for one input and one algorithm

• But impossible to solve for all inputs and algorithms
Remarks

• Easy to solve for one input and one algorithm

• But impossible to solve for all inputs and algorithms

• Result holds for all computational models
Remarks

• Easy to solve for one input and one algorithm

• But impossible to solve for all inputs and algorithms

• Result holds for all computational models

• All non-trivial properties of algorithms are undecidable
Compiler
• Takes

COMPILER

• Takes
COMPILER

• Takes
  • String $A$ describing algorithm
  • String $x$ describing algorithm’s input
• Takes
  • String $A$ describing algorithm
  • String $x$ describing algorithm’s input
• Outputs $A(x)$
COMPILER

• Takes
  • String A describing algorithm
  • String x describing algorithm’s input
• Outputs A(x)

• Compiler itself is an algorithm, too!
Function $A_{\text{diag}}(x)$ is defined as follows:
UNDECIDABLE PROBLEM

• Function $A_{\text{diag}}(x)$ is defined as follows

• If the algorithm $x$ on input $x$ outputs 1, then $A_{\text{diag}}(x) = 0$

• If the algorithm $x$ on input $x$ outputs other value or never halts, then $A_{\text{diag}}(x) = 1$
• Function $A_{\text{diag}}(x)$ is defined as follows

• If the algorithm $x$ on input $x$ outputs 1, then $A_{\text{diag}}(x) = 0$

• If the algorithm $x$ on input $x$ outputs other value or never halts, then $A_{\text{diag}}(x) = 1$
DIAGONALIZATION
REDUCTION FROM DIAG TO HALT

• Assume there exists an algorithm for HALT
Reduction from Diag to HALT

• Assume there exists an algorithm for HALT

• Given input $x$, we check if the algorithm $x$ halts on $x$
Reduction from Diag to HALT

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• Given input $x$, we check if the algorithm $x$ halts on $x$

• If it doesn’t halt, output 1
Reduction from Diag to HALT

- Assume there exists an algorithm for HALT

- Given input \( x \), we check if the algorithm \( x \) halts on \( x \)

- If it doesn’t halt, output 1

- If it halts and outputs 1, output 0
REDUCTION FROM DIAG TO HALT

• Assume there exists an algorithm for HALT

• Given input $x$, we check if the algorithm $x$ halts on $x$

• If it doesn’t halt, output 1

• If it halts and outputs 1, output 0

• If it halts and outputs something else, output 1